Testing Hypotheses

- Overview
- 5 Steps for testing hypotheses
- Research and null hypotheses
- One and two-tailed tests
- Type 1 and Type 2 Errors
- Z tests and t tests

Testing Hypotheses
(and Null Hypotheses)

Testing Hypotheses is a procedure that allows us to evaluate hypotheses about population parameters based on sample statistics.

Example of an hypothesis:

Sociology undergraduates at UNT have a higher average GPA score than all other undergraduates at UNT.

Research Hypothesis
(and Null Hypothesis)

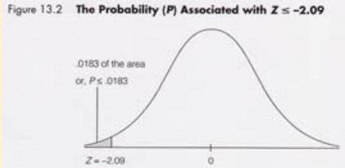
A research hypothesis (H_1) is a statement reflecting a substantive hypothesis (i.e., the stated relationship between two population parameters).

A null hypothesis (Ho) is a statement of "no difference" that is in opposition to the research hypothesis (for example: the average *GPA* score of sociology undergraduates at UNT is no different than that of other students at UNT).

One-Tailed Tests

One-tailed hypothesis test – A hypothesis test in which the population parameter is known to fall to the right or the left of center of the normal curve.

Figure 13.2 **The Probability (P) Associated with $Z \le -2.09$**



One-Tailed Tests

- **Right-tailed test** – A one-tailed test in which the sample statistic is hypothesized to be at the right tail of the sampling distribution.
- **Left-tailed test** – A one-tailed test in which the sample statistic is hypothesized to be at the left tail of the sampling distribution.

Two-Tailed Hypothesis Test

A hypothesis test in which a parameter statistic might fall within either the right or left tail of the sampling distribution (we are not sure which tail of the curve the statistic is likely to fall).

The Five Steps In Hypothesis Testing

1. Making assumptions about the data

--a random sample is being used

--knowing the level of measurement of the data, in the examples that we will be using, we will assume the dependent variable is interval/ratio

--either the variable is normally distributed or the sample is over 50 cases, this will allow us to apply the Central Limit Theorem

The Five Steps In Hypotheses Testing

2. Stating the **research** and **null hypotheses** and selecting **alpha**.

Research hypothesis (H_1) – A statement reflecting the substantive hypothesis. The research hypothesis is always expressed in terms of population parameters.

Null hypothesis (H_0) – A statement of "no difference," which contradicts the research hypothesis and is always expressed in terms of population parameters.

The Five Steps In Hypotheses Testing

2. (Continued) Stating the **research** and **null hypotheses** and selecting **alpha**.

Research hypothesis (H_1) – For example, we might state that the average salary of women in the population is less than that of those in the general population (general population = $28,985).

H_1 : Uy < $28,985

The Five Steps In Hypotheses Testing

2. (Continued) Stating the **research** and **null hypotheses** and selecting **alpha**.

Null hypothesis (H_0) – The null hypothesis would state that there is no difference between the salary of women and the salary of those in the general population.

H_o : Uy = $28,985

The Five Steps In Hypotheses Testing

2. (Continued) Stating the **research** and **null hypotheses** and selecting **alpha**.

Alpha (α) – Is the level of probability at which the null hypothesis is rejected. We decide where we want to set alpha. It is customary to set alpha at the .05, .01, or .001 level.

The Five Steps In Hypotheses Testing

2. (Continued) Stating the research and null hypotheses and selecting alpha.

Type I error: if the null hypothesis is true but we reject it.

Type II error: if the null hypothesis is false but we accept it.

Type I and Type II Errors and their relationship to alpha

- During this step we need to be aware that if we set **alpha too large** (e.g. .10) we may create a Type I error—that is, we might reject the null hypothesis when it is actually true.
- Or, if we set the **alpha too small** (e.g., .001) we may create a **Type II** error by failing to reject a false null hypothesis.

Type I and Type II Errors

Based on sample results, the decision made is to…

In the population H_0 is …		**reject H_0**	**do not reject H_0**
	true	**Type I error**	correct decision
	false	correct decision	**Type II error**

The Five Steps In Hypotheses Testing

3. Selecting the **sampling** distribution and specifying the test statistic

   --to test the null hypothesis we sample at least 50 cases so that our theoretical sampling distribution will be normally distributed.

   --The test statistic used is either the **Z statistic or t statistic**

The Five Steps In Hypotheses Testing

4. **Computing the test statistic**. The formula for the Z statistic is:

$$Z = \frac{\bar{Y} - u_y}{\frac{\sigma_y}{\sqrt{N}}} \quad \text{or} \quad \frac{\text{Group Mean – Population Mean}}{\frac{\text{Population SD}}{\sqrt{N}}}$$

The Five Steps In Hypotheses Testing

4. **Computing the test statistic**. The formula for the Z statistic is:

$$Z = \frac{\bar{Y} - u_y}{\frac{\sigma_y}{\sqrt{N}}} \quad \text{or} \quad \frac{24{,}100 - 28{,}985}{\frac{23{,}335}{\sqrt{100}}} = -2.09$$

Where the population mean is $28,985 and the sample mean for women is $24,100 with a standard deviation of 23,335 and sample size of 100

The Five Steps In Hypotheses Testing: Probability Values

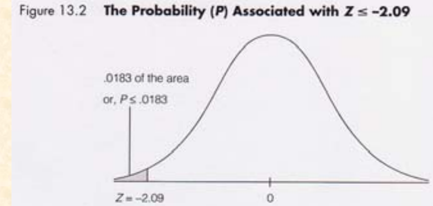


Figure 13.2 **The Probability (P) Associated with $Z \leq -2.09$**

24,100 Women

28,985 Whole Population

When to use the t statistic and when to use the Z statistic

1. The Z statistic can only be used if the **population standard deviation** is known. Typically, this is not the case.

2. When the **sample standard deviation** must be used in lieu of the population SD then the t statistic should be used.

3. The formula for the t statistic is identical to the formula for the Z statistic except that **the sample SD is used in place of the population SD**

The Five Steps In Hypotheses Testing

4. **Computing the test statistic.** The formula for the t statistic is:

$$t = \frac{\overline{Y} - u_Y}{\frac{S_Y}{\sqrt{N}}} \quad \text{or} \quad \frac{\text{Group Mean} - \text{Population Mean}}{\frac{\text{Sample SD}}{\sqrt{N}}}$$

The Five Steps In Hypotheses Testing

5. **Making a Decision and Interpreting** the results. In our example:

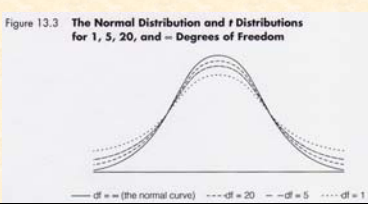
--we confirm that the Z is on the left tail of the distribution (-2.09)

--the **P value** found in the Z table (where Z = 2.09) is .0183, which is less than a .05 **alpha.**

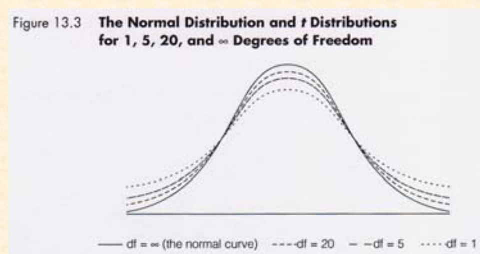
--thus, we can reject the null hypothesis of no difference and can conclude that the average income of the general population is greater than that of women

Interpretation of the t statistic and the Degrees of Freedom

1. The t statistic has its own t distribution table that is used rather than the Z distribution table. This is because the Z distribution always assumes a normal curve while the curve of the t distribution varies somewhat depending on the size of the sample.



t distribution -a smaller degree of freedom produces a flatter curve



Interpretation of the t statistic and the Degrees of Freedom

2. Reading the t distribution table requires knowing the **degrees of freedom** (a concept used in calculating a number of statistics including the t statistic).

   --Reading the t distribution table also requires knowing the **alpha** (which you select) and the **number of cases.**

Interpretation of the t statistic and the Degrees of Freedom

3. The **degrees of freedom** represent the number of scores that are free to vary in calculating each statistic.

4. Typically the **degrees of freedom** are N – 1 when comparing a group to a whole population.

Using the t statistic

- In our previous example we knew that:

  The population mean is $28,985 and the sample mean for women is $24,100 with a population standard deviation of 23,335 and sample size of 100.
- We subsequently calculated the Z score.
- If we did not know the population SD, we would need to use the sample SD (which is $24,897) and then calculate the "t" score.

For Example:

The population mean is $28,985 and the sample mean for women is $24,100 with a sample standard deviation of 24,897 and sample size of 100.

$$\frac{24{,}100-28{,}985}{\frac{24{,}897}{\sqrt{100}}} = \frac{-4885}{2489.7} = -1.96$$

Degrees of Freedom

- Our **degrees of freedom** for this example is N – 1 or 99 and our t statistic is -1.96 (the larger the t statistic the more likely it will be significant).
- On page ??? of your book we can find the **t distribution table**. It displays the **degrees of freedom** for 60 and for 120 (or see next slide). Since ours is 99 it falls between these.
- We can assume a **one-tailed test** since existing knowledge indicates that women make less than the population as a whole and certainly not more (the mean will fall on the left side of the curve).

t distribution table

Table 13.2 **Values of the *t* Distribution**

	Level of Significance for One-Tailed Test					
	.10	.05	.025	.01	.005	.0005
	Level of Significance for Two-Tailed Test					
df	.20	.10	.05	.02	.01	.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.598
3	1.638	2.353	3.182	4.541	5.841	12.941
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.859
10	1.372	1.812	2.228	2.764	3.169	4.587
15	1.341	1.753	2.131	2.602	2.947	4.073
20	1.325	1.725	2.086	2.528	2.845	3.850
25	1.316	1.708	2.060	2.485	2.787	3.725
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.291

Source: Abridged from R. A. Fisher and F. Yates, *Statistical Tables for Biological, Agricultural and Medical Research*, Table 111. Copyright © R. A. Fisher and F. Yates 1963. Reprinted by permission of Pearson Education Limited.

Degrees of Freedom

- From the **t distribution table** we can conclude that our sample mean is statistically significant from the general population mean because:

1. On page ??? of your book we can see that, for 60 **degrees of freedom**, a t statistic of 1.671 has a **p value** of .05 when using a one-tailed test. This is large enough to be statistically significant at **alpha** .05.

2. the **degrees of freedom** in our analysis is 99, therefore if our t statistic is 1.671 or larger we can conclude that our sample mean is statistically significant with a confidence level of at least 95%.

Degrees of Freedom

3. Since our actual t statistic is -1.96 we can conclude statistical significance at the .05 level.

4. We have assumed a **one-tailed test** since existing knowledge indicates that women make less than the population as a whole and certainly not more. **If we did not know whether women make more or less than men we would need to use a two tailed test**.

t Test in Sum

- **t statistic** – The test statistic computed to test the null hypothesis about a population mean when the population standard deviation is unknown and is estimated using the sample standard deviation (note: this is not the only situation where the t statistic is used, we will learn others).
- **t distribution** – A family of curves, each determined by its degrees of freedom (df). It is used when the population standard deviation is unknown and the standard error is estimated from the sample standard deviation.
- **Degrees of freedom *(df)*** – The number of scores that are free to vary in calculating a statistic.

Summary: Steps in Testing an Hypothesis

1. Verify assumptions are met
2. State research and null hypotheses
3. Select sampling distribution and test statistic (Z or t statistic)
4. Compute test statistic
5. Make a decision and interpret results

Comparing Two Sample Means

(Rather than a Sample and a Population as just learned)

Example for comparing two means:

– Comparing the mean salary for women to the mean salary for men (instead of the mean salary of women to the mean salary of the whole population)

Comparing Two Sample Means

(Rather than a Sample and a Population as just learned)

Steps for comparing two sample statistics are the same as those for comparing a sample statistic to a population parameter except for:

the formula for calculating
(1) the t statistic and
(2) the degrees of freedom

Calculating the **t statistic:**

$$\frac{\text{Mean of 1}^{\text{st}}\text{ Group} \; - \; \text{Mean of 2}^{\text{nd}}\text{ Group}}{\text{Standard Error of the Differences Between the Means}}$$

Calculating t Statistic

$$\frac{\text{Mean of } 1^{st} \text{ Group} \quad - \quad \text{Mean of } 2^{nd} \text{ Group}}{\textbf{Standard Error} \text{ of the Differences Between the Means}}$$

$$SE = \sqrt{\frac{(N-1)\ S1y1 \quad - \quad (N2-1)\ S2y2}{(N1 + N2) - 2}} \sqrt{\frac{N1+N2}{N1N2}}$$

Where:

N1 = Number of cases for 1^{st} group

N2 = Number of cases for 2^{nd} group

S1y1 = Sample variance for 1^{st} group

S2y2 = Sample variance for 2^{nd} group

Calculating Degrees of Freedom

$$df = (N1 + N2) -2$$

Your text provides an example for calculating the t when comparing two means.